Initial states of qubit–boson models leading to conserved quantities

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There are initial conditions of composite qubit–environment systems which provide conservation of certain qubit observables but are not associated with a conservation law related to a symmetry of the system. This states are shown to form a subspace of the qubit–environment space of states. General construction of such states is presented and illustrated by two examples: The first one is exactly solvable Jaynes–Cummings model and the second is the multi–photon Rabi model.

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Introduction. Interaction of two quantum systems almost always modify a set of quantum numbers suitable for the description of non–interacting components. Generically, only in the absence of interactions the set of good quantum numbers of the composite system consists of good quantum numbers of its constituents. Let us consider the energy as an example. It is known that (Hamiltonian–type) interaction of two quantum systems causes an energy flow from one system to the other and/or vice–versa. The total system remains conservative but the character of the subsystems changes qualitatively. They become *open* [1]. Interestingly, there is a very specific situation, commonly refereed to as *dephasing* (or pure decoherence) [2, 3], when the energy exchange is absent during the entire evolution, regardless of the initial conditions. However, for such phenomenon to occur one requires a rather uncommon feature of the interaction resulting in the existence of conservation law(s). It is also the case if, instead of the energy, one considers any observable acting on the constituent of the total system. One cannot expect its conservation unless certain symmetry is present in the system.

At this point one can pose a natural question: Is it possible to design quantum evolution that guarantees conservation of, at least, some observables regardless of the existence of obvious symmetries in the system? In this Brief Report we provide the answer in the case of two–level quantum systems interacting with the environment.

The layout of this work is as follows. We begin by showing how a proper choice of an initial preparation of the composite qubit–boson system can assures no energy exchange between the subsystems. In other words, we identify dephasing–like behaviour in non–dephasing model. Next, in section II a general method is presented. Section III followed by the conclusions IV serves as our second example.

First example. Let us start with a simple exactly solvable Jaynes–Cummings model [4]:

$$\mathrm{H} = \omega\sigma_z + \nu a^\dagger a + \left(g^*\sigma_+ \otimes a + g\sigma_- \otimes a^\dagger\right), \tag{1}$$

where a and $a^\dagger$ are the creation and annihilation operators of the bosonic field (environment) *i.e.*, $[a, a^\dagger] = \mathbb{I}_\mathrm{B}$ and σ_z denotes the one of the Pauli spin matrices with eigenstates $|\pm\rangle$, whereas ν and ω describe the qubit and field energies, respectively. The coupling constant g reflects the strength of the interaction between the systems, while $\sigma_\pm|\mp\rangle = |\pm\rangle$ and $\sigma_\pm|\pm\rangle = 0$ are ladder operators.

Although the model (1) essentially originates from quantum optics [4], it has been studied in wide range of other branches of physics (see *e.g.* [5]), for almost half–century, within the broad variety of contexts [6]. Therefore, it serves as a good starting point for our considerations.

In this example we are interesting in finding qubit–boson initial state(s) ϱ, such that the qubit energy $E_\mathrm{Q} = \omega\mathrm{Tr}\left(\sigma_z \otimes \mathbb{I}_\mathrm{B}\varrho(t)\right) \sim \langle\sigma_z\rangle$ is conserved, or $E_\mathrm{Q} = \omega\mathrm{Tr}\left(\sigma_z\rho(t)\right) = cst.$ where the reduced qubit dynamics $\rho(t) = \mathrm{Tr}_\mathrm{B}(e^{-i\mathrm{H}t}\varrho e^{i\mathrm{H}t})$ is used [7].

It is well known that for the Jaynes–Cummings model the total number of excitation $\mathrm{N} = a^\dagger a + \sigma_z$ is conserved which ultimately leads to the exact diagonalization of the model and in particular results in exact solvability of its dynamics. At this stage we want to emphasize that none of the above features is essential in the following analysis, yet involving N one simplifies calculations.

We start by splitting the Hamiltonian (1) into two commuting parts, $\mathrm{H} = \mathrm{H}_0 + \mathrm{V}$, where $\mathrm{H}_0 = \nu\mathrm{N}$ and

$$\mathrm{V} = \delta\sigma_z + \left(g^*\sigma_+ \otimes a + g\sigma_- \otimes a^\dagger\right), \tag{2}$$

where $\delta \equiv \omega - \nu$ is the detuning frequency. As the result the time evolution of the systems can be factorized such that

$$\mathrm{U}_t = \exp(-i\mathrm{H}_0 t)\exp(-i\mathrm{V}t) \equiv \mathrm{U}_t^0\mathrm{V}_t. \tag{3}$$

Let us observe that in this example $\rho(t)$ mimics the pure dephasing evolution if

$$\begin{aligned}\rho(t) &= \alpha|+\rangle\langle+| + c(t)^*|-\rangle\langle+| \\ &+ c(t)|+\rangle\langle-| + (1-\alpha)|-\rangle\langle-|,\end{aligned} \tag{4}$$

where α is a real constant and $c(t)$ denotes a function of time such that $|c(t)| \leq 1$.

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At this point two questions can be addressed: Which initial states of the composite system, ϱ, do guarantee the reduced dynamics (4)? Are they separable or entangled?

In order to answer this questions we offer an explicit construction of such states. At face value subsequent steps may seem bizarre and to some extent artificial. Notwithstanding this, they can be performed, as it is discussed below, for a broad class of qubit–boson models. As a first step we define $\varrho = |\Psi\rangle\langle\Psi|$, where

$$|\Psi\rangle = C_\psi \left(|+\rangle \otimes |\psi\rangle + |-\rangle \otimes \mathrm{X}|\psi\rangle\right), \tag{5}$$

with $|\psi\rangle$ an arbitrary state of the bosonic field. C_ψ is a normalization constant and $\langle\Psi|\Psi\rangle = 1$. For the sake of simplicity we absorb it into $|\psi\rangle$ so that $\|\psi\| = |C_\psi|$.

Possibility of constructing dephasing states relies on the ability of finding a linear operator X satisfying

$$g^*\mathrm{X}a\mathrm{X} + 2\delta\mathrm{X} - ga^\dagger = 0, \tag{6}$$

or $\mathrm{X}(\delta + g^*a\mathrm{X}) = -\delta\mathrm{X} + ga^\dagger$. As it will be seen, this choice of X allows for a specific time evolution of the above state, namely

$$|\Psi\rangle \to |\Psi_t\rangle = \mathrm{U}_t^0 \left(|+\rangle \otimes |\psi_t\rangle + |-\rangle \otimes \mathrm{X}|\psi_t\rangle\right). \tag{7}$$

According to (7), the dynamics of the entire system consists of two parts. The one governed by $\mathrm{U}_t^0 = \exp(-i\mathrm{H}_0 t)$ is the free evolution. The other one, encoded in $|\psi_t\rangle$, is the remaining part resulting from the interaction and it needs to be determined along with X. This results in the following reduced dynamics

$$\begin{aligned} \alpha(t) &= \langle\psi_t|\psi_t\rangle, \\ c(t) &= e^{-i2\nu t}\langle\psi_t|e^{i\nu a^\dagger a t}\mathrm{X}e^{-i\nu a^\dagger a t}|\psi_t\rangle, \end{aligned} \tag{8}$$

which resembles the dephasing dynamics (4) if and only if the vector $|\psi_t\rangle$ itself evolves unitarily. For this to hold true there must exist a Kamiltonian K such that $|\psi_t\rangle = e^{-i\mathrm{K}t}|\psi\rangle$. In order to see this is indeed the case note that according to (6) and (2), for $\mathrm{K} = \delta + g^*a\mathrm{X}$ one has

$$\begin{aligned} \mathrm{V}|\Psi\rangle &= |+\rangle \otimes \mathrm{K}|\psi\rangle + |-\rangle \otimes \left(-\delta\mathrm{X} + ga^\dagger\right)|\psi\rangle \\ &= |+\rangle \otimes \mathrm{K}|\psi\rangle + |-\rangle \otimes \mathrm{XK}|\psi\rangle, \end{aligned} \tag{9}$$

for every $|\psi\rangle$. Thus, by replacing $|\psi\rangle$ with $\mathrm{K}|\psi\rangle$ one finds

$$\mathrm{V}^2|\Psi\rangle = |+\rangle \otimes \mathrm{K}^2|\psi\rangle + |-\rangle \otimes \mathrm{XK}^2|\psi\rangle \tag{10}$$

and by repeating this procedure n times one obtains

$$\mathrm{V}^n|\Psi\rangle = |+\rangle \otimes \mathrm{K}^n|\psi\rangle + |-\rangle \otimes \mathrm{XK}^n|\psi\rangle. \tag{11}$$

Therefore

$$\begin{aligned} \mathrm{V}_t|\Psi\rangle &= \sum_{n=0}^{\infty} \frac{(-it)^n}{n!}\mathrm{V}^n|\Psi\rangle \\ &= |+\rangle \otimes e^{-i\mathrm{K}t}|\psi\rangle + |-\rangle \otimes \mathrm{X}e^{-i\mathrm{K}t}|\psi\rangle. \end{aligned} \tag{12}$$

It remains 'only' to identify X and to show that K is Hermitian. There is no general method allowing for finding solutions to quadratic equations like Eq. (6), yet judging from its structure it seems reasonable to try $\mathrm{X} = \sum_n \xi_n|n+1\rangle\langle n|$. By explicit substitution one infers that this is indeed a good guess as long as

$$\xi_n = \frac{-\delta + \sqrt{\delta^2 + |g|^2(n+1)}}{g^*\sqrt{n+1}}, \quad n \geq 0. \tag{13}$$

The solution we have just found generalizes the Susskind–Glogower operator [8], in the sense that $\mathrm{X} \to (aa^\dagger)^{-1/2}a^\dagger$ when $\delta \to 0$. As the result $\mathrm{K} = \sqrt{\delta^2 + |g|^2(a^\dagger a + \mathbb{I}_\mathrm{B})}$ and it clearly is Hermitian. From Eq. (8) follows $\alpha = \langle\psi|\psi\rangle$ and in addition to that

$$c(t) = e^{-i\nu t}\sum_{n=0}^{\infty} \xi_n e^{i\Omega_n t}\langle\psi|n+1\rangle\langle n|\psi\rangle, \tag{14}$$

where $\Omega_n = \sqrt{\delta^2 + |g|^2(n+2)} - \sqrt{\delta^2 + |g|^2(n+1)}$.

Note, we still have the freedom to choose the vector $|\psi\rangle$. In particular, putting $|\psi\rangle \sim |m\rangle$, where $|m\rangle$ is a state with defined number of bosons, we have not only $\dot{c}(t) = 0$, meaning $\rho(t)$ is a steady state, but also $c(t) = 0$ *i.e.* it exhibits its classical nature.

In general, states alike the one from Eq. (5) are entangled, hence there is no one–to–one correspondence between qubit–boson density operators $|\Psi\rangle\langle\Psi|$ and reduced density matrices $\mathrm{Tr}_\mathrm{B}(|\Psi\rangle\langle\Psi|)$ of the qubit [9]. However, if $|\psi\rangle$ is an eigenvector of X, *i.e.* $\mathrm{X}|\psi\rangle = \lambda|\psi\rangle$, $\lambda \in \mathbb{C}$, then $|\Psi\rangle = (|+\rangle + \lambda|-\rangle) \otimes |\psi\rangle$ is separable.

Generalization. So far we have showed how to determine initial preparations of the specific composite qubit–boson system which guarantee no energy flow between qubit and its environment (boson), *i.e.* the reduced qubit dynamics mimics pure dephasing evolution. We also investigated conditions under which from this broad class of states one can chose the separable ones.

Now, we are interested in finding answer to a more general question: How to prepare initial state(s) of the composite system (and determine their separability), which result in no 'information flow' between the subsystems, reflected in changes of a qubit observable Λ. Such states have very much in common with dephasing states and for that reason we will keep this terminology.

Previously, we have conducted our analysis by making use of a very specific exactly solvable model. Currently, we will show that neither solvability nor the form of the interaction in this model is a *sine qua non* condition for building 'dephasing states' for general qubit–environment models.

Let Λ be a 2×2 Hermitian matrix–a given qubit observable. Our objective is to determine $\rho \equiv \rho(0)$ such that $\langle\Lambda(t)\rangle \equiv \mathrm{Tr}\left(\Lambda\rho(t)\right)$ remains constant during the evolution. As before, by $\rho(t)$ we denote the qubit reduced dynamics, $\rho(t) = \mathrm{Tr}_\mathrm{E}\left[e^{-i\mathbf{H}t}|\Psi\rangle\langle\Psi|e^{i\mathbf{H}t}\right]$, where $|\Psi\rangle$ is the initial qubit–environment state and

$$\mathbf{H} = \mathrm{H}_\mathrm{Q} \otimes \mathbb{I}_\mathrm{E} + \mathbb{I}_\mathrm{Q} \otimes \mathrm{H}_\mathrm{E} + \mathbf{H}_\mathrm{int}, \tag{15}$$

with all the symbols having their usual meaning, stands for the total Hamiltonian.

We begin with a very simple observation that in each moment of time t and for every complex 2×2 matrix Λ, the partial trace, $\mathrm{Tr}_{\mathrm{E}}(\cdot)$, satisfies $\mathrm{Tr}\left[\Lambda \mathrm{Tr}_{\mathrm{E}}(\varrho(t))\right] = \mathrm{Tr}\left[(\Lambda\otimes\mathbb{I}_{\mathrm{E}})|\Psi(t)\rangle\langle\Psi(t)|\right]$. By virtue of this relation

$$\langle\Lambda(t)\rangle = \mathrm{Tr}\left[(\Lambda_d\otimes\mathbb{I}_{\mathrm{E}})\,|\Omega(t)\rangle\langle\Omega(t)|\right], \tag{16}$$

with $|\Omega(t)\rangle = e^{-i\mathbf{K}t}|\Omega\rangle$, where $|\Omega\rangle = \mathrm{U}\otimes\mathbb{I}_{\mathrm{E}}|\Psi\rangle$,

$$\mathbf{K} = (\mathrm{U}\otimes\mathbb{I}_{\mathrm{E}})\mathbf{H}(\mathrm{U}\otimes\mathbb{I}_{\mathrm{E}})^\dagger \tag{17}$$

and U denotes the unitary matrix such that $\mathrm{U}^\dagger\Lambda\mathrm{U} = \Lambda_{\mathrm{d}}$, with $\Lambda_{\mathrm{d}}\equiv\mathrm{diag}(\lambda_+,\lambda_-)$. The Kamiltonian $\mathbf{K}$ can always be written as

$$\begin{aligned}\mathbf{K} = {}&|+\rangle\langle+|\otimes\mathrm{H}_+ + |-\rangle\langle-|\otimes\mathrm{H}_-\\ &+|+\rangle\langle-|\otimes\mathrm{V} + |-\rangle\langle+|\otimes\mathrm{V}^\dagger.\end{aligned} \tag{18}$$

An explicit form of $\mathrm{H}_\pm$ and V can easily be recovered when the Hamiltonians H_{Q}, H_{E}, and $\mathbf{H}_{\mathrm{int}}$ are provided. *A priory* we neither impose any physical restriction of their specification nor assume existence of symmetries in the total system (resulting *e.g.*, in its solvability).

Initially the composite system is assumed to be in the state $\varrho = |\Psi\rangle\langle\Psi|$, where

$$|\Psi\rangle = C_\psi\left(|\lambda_+\rangle\otimes|\psi\rangle + |\lambda_-\rangle\otimes\mathrm{X}|\psi\rangle\right), \tag{19}$$

with $|\lambda_\pm\rangle = \mathrm{U}^\dagger|\pm\rangle$. $|\psi\rangle$ is a freely chosen state of the environment. As before, one can redefine the state $|\psi\rangle$ so that $|\psi\rangle\to C_\psi|\psi\rangle$. We show that $|\Omega\rangle$ undergoes the following evolution

$$|\Omega\rangle\to e^{-i\mathbf{K}t}|\Omega\rangle = |+\rangle\otimes|\psi_t\rangle + |-\rangle\otimes\mathrm{X}|\psi_t\rangle, \tag{20}$$

provided that X solves the (operator) Riccati equation:

$$\mathrm{XVX} + \mathrm{XH}_+ - \mathrm{H}_-\mathrm{X} - \mathrm{V}^\dagger = 0, \tag{21}$$

where $|\psi_t\rangle = e^{-i\mathrm{K}_+t}|\psi\rangle$, for some operator K_+. Riccati equations of the type (21) have been studied not only in the broad mathematical context [10] but also have recently been applied to investigations of open quantum systems [11]. Let us also notice here that Eq. (21) reduces to the condition (6) when $\mathbf{H}$ is given by (1), as one may anticipate.

Now, let $\mathrm{K}_+ = \mathrm{H}_+ + \mathrm{VX}$, then from (21) immediately follows that $\mathrm{XK}_+ = \mathrm{H}_-\mathrm{X} + \mathrm{V}^\dagger$, which leads to

$$\mathbf{K}|\Omega\rangle = |+\rangle\otimes\mathrm{K}_+|\psi\rangle + |-\rangle\otimes\mathrm{XK}_+|\psi\rangle. \tag{22}$$

As before one can easily justify the general formula:

$$\mathbf{K}^n|\Omega\rangle = |+\rangle\otimes\mathrm{K}_+^n|\psi\rangle + |-\rangle\otimes\mathrm{XK}_+^n|\psi\rangle, \quad n\geq 0, \tag{23}$$

from which the evolution generated by $\mathbf{K}$ is easily obtained to be

$$\begin{aligned}|\Omega(t)\rangle &= \sum_{n=0}^{\infty}\frac{(-it)^n}{n!}\mathbf{K}^n|\Psi\rangle\\ &= |+\rangle\otimes e^{-i\mathrm{K}_+t}|\psi\rangle + |-\rangle\otimes\mathrm{X}e^{-i\mathrm{K}_+t}|\psi\rangle.\end{aligned} \tag{24}$$

Having (24) in place, one can formulate conditions under which (16) does not depend on time. A necessary condition for such evolution is for XV to be an observable. Because then K_+ is Hermitian and hence $|\psi_t\rangle$ evolve in unitary fashion. It is a matter of straightforward calculation to show that in this case $\mathrm{Tr}_{\mathrm{E}}(|\Omega(t)\rangle\langle\Omega(t)|)$ has the dephasing structure (4), where

$$\alpha = \langle\psi|\psi\rangle, \quad c(t) = \langle\psi|e^{i\mathrm{K}_+t}\mathrm{X}e^{-i\mathrm{K}_+t}|\psi\rangle, \tag{25}$$

and therefore from (16) follows $\langle\Lambda(t)\rangle = \alpha\lambda_+ + (1-\alpha)\lambda_-$.

Interestingly, the dephasing dynamics can also be designed by starting from any state orthogonal to (19). Indeed, such states are found to be of the form:

$$|\Phi\rangle = C_\phi\left(|\lambda_-\rangle\otimes|\phi\rangle - |\lambda_+\rangle\otimes\mathrm{X}^\dagger|\phi\rangle\right), \tag{26}$$

which can be verified by noticing that $\langle\Psi|\Phi\rangle = 0$, for every $|\psi\rangle, |\phi\rangle\in\mathcal{H}_{\mathrm{E}}$. In this case the dephasing dynamics takes the form

$$\alpha = 1 - \langle\phi|\phi\rangle, \quad c(t) = \langle\phi|e^{i\mathrm{K}_-t}\mathrm{X}^\dagger e^{-i\mathrm{K}_-t}|\phi\rangle, \tag{27}$$

provided $\mathrm{K}_- = \mathrm{H}_- - \mathrm{V}^\dagger\mathrm{X}^\dagger$ is Hermitian. For instance, in the Jaynes–Cummings model discussed in the preceding section we find that $\mathrm{K}_- = \sqrt{\delta^2 + |g|^2a^\dagger a}$.

Second example. As a second example we consider the k-photon Rabi model [12], for which $\mathrm{H}_{\mathrm{Q}} = \omega\sigma_z$, $\mathrm{H}_{\mathrm{E}} = \nu a^\dagger a$ read exactly as in the James–Cummings model (1) but the interaction is given by

$$\mathbf{H}_{\mathrm{int}} = \sigma_x\otimes(g^*a^k + g(a^\dagger)^k). \tag{28}$$

This model not only generalizes the single mode case but also includes counter-rotating-wave terms [13], which make its analytical treatment much more complicated in compare with (1). Despite recent progress in finding it analytic solution [14], the problem remains highly non–trivial.

Let us suppose that this time we want to find initial state(s) of the composite system such that the x–component of the qubit spin operator remains constant during the evolution *i.e.*, $S_x = \frac{1}{2}\mathrm{Tr}\,(\sigma_x\rho(t)) = cst$. First, one needs to determine U which transforms σ_x to its diagonal form. It is an easy task to do and the answer is $\mathrm{U} = (\sigma_z + \sigma_x)/\sqrt{2}$. In this case we have $\mathrm{U}\sigma_x\mathrm{U}^\dagger = \sigma_z$, that is $\lambda_\pm = \pm 1$. Next, we transform $\mathbf{H}$ into $\mathbf{K}$ according to (17) and then recover $\mathrm{H}_\pm$, V from the decomposition (18). Straightforward calculation shows

$$\mathrm{H}_\pm = \nu a^\dagger a \pm\left(g^*a^k + g(a^\dagger)^k\right), \quad \mathrm{V} = \omega\mathbb{I}_{\mathrm{E}}. \tag{29}$$

The corresponding Riccati equation (21) reads as follows

$$\omega\mathrm{X}^2 + \mathrm{XH}_+ - \mathrm{H}_-\mathrm{X} - \omega\mathbb{I}_{\mathrm{E}} = 0. \tag{30}$$

We can solve this equation by introducing the generalize parity operator [15]:

$$\mathrm{X}_k = \sum_{l=1}^{k}\sum_{n=0}^{\infty}(-1)^n|n,l\rangle\langle n,l|, \tag{31}$$

where $|n,l\rangle := |kn+l-1\rangle$ and $l \leq k$. Such operator is both Hermitian and unitary, hence also $\mathrm{X}_k^2 = \mathbb{I}_\mathrm{E}$. Note that for $k=1$ this parity simplifies to the well know bosonic parity operator $\mathrm{P} = \exp(i\pi a^\dagger a)$. As one can verify $\mathrm{X}_k a^k \mathrm{X}_k = -a^k$, thus $\mathrm{X}_k \mathrm{H}_+ \mathrm{X}_k = \mathrm{H}_-$ and therefore X_k indeed solves (30). This fact may come as a surprise since (31) does not depend on any of the parameters ν, ω, g. In this model the dephasing dynamics (25) and (27) are generated by $\mathrm{K}_\pm = \mathrm{H}_\pm \pm \omega \mathrm{X}_k$, respectively.

Introducing projections $\mathrm{P}_\pm = \frac{1}{2}(\mathbb{I}_\mathrm{E} \pm \mathrm{X}_k)$, onto subspaces $\mathcal{H}_\pm$ consist of states with defined parity (with respect to the generalized parity (31)) and taking into account both (5) and (26) we have

$$|\Psi_\epsilon\rangle = \frac{1}{2}\left(|+\rangle \otimes \mathrm{P}_\epsilon|\psi\rangle + |-\rangle \otimes \mathrm{P}_{-\epsilon}|\psi\rangle\right), \quad \epsilon = \pm 1, \quad (32)$$

which are separable if $|\psi\rangle \in \mathcal{H}_\pm$.

Summary. The more conservation laws are present in the system (either classical or quantum) the better is our understanding of its behaviour and properties. In this Brief Report we have presented a method of finding time evolutions of quantum systems guaranteeing conservation of certain quantities in the absence of related conservation laws. This is done by a proper choice of initial states.

In order to built initial states resulting in the observable–conserving evolution one needs to solve the corresponding Riccati equation. Unfortunately, neither the form nor even the existence of such a solution can be taken for granted in general. Nevertheless, some useful criteria for solvability, applicable to the broad range of relevant physical systems, can be found in literature [16].

Preparation of the observable–conserving states $|\Psi\rangle$ or $|\Phi\rangle$ would require highly sophisticated quantum engineering. One could construct a state $|\chi\rangle = \mathrm{X}|\psi\rangle$, then 'tensorize' the result with a qubit state $|-\rangle \otimes |\chi\rangle$, and finally superpose with the tensorized qubit–environment state $|+\rangle \otimes |\psi\rangle$. Clearly, the situation becomes simpler if the states are separable as it is the case for the broad class of problems when $|\psi\rangle$ is an eigenstate of X.

Despite obvious difficulties there is also at least one advantage of this procedure: It is performed only once at the beginning of the evolution–no dynamical control is needed to maintain the desired dynamics. We hope that together with continuous development of the quantum state engineering techniques the construction proposed in this work can become useful for applications.

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